Econ 802

Final Exam

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Read each question carefully and try to use all of the information provided. All questions have equal weight. If something is unclear, please ask.

1. There are firms $j = 1 \ldots m$ with production possibility sets Y_j. The price vector is $p = (p_1 \ldots p_k)$ and firm j's production plan is $y_j = (y_{j1} \ldots y_{jk})$. Outputs have positive signs and inputs have negative signs. Define the aggregate production possibility set Y as follows: $y \in Y$ if and only if it is possible to write $y = y_1 + y_2 + \ldots + y_m$ where $y_j \in Y_j$ for all $j = 1 \ldots m$.

(a) Assume the profit functions $\pi_j(p) = \max py_j$ subject to $y_j \in Y_j$ are well defined for all j. Prove that the aggregate profit function $\pi(p) = \max py$ subject to $y \in Y$ must satisfy $\pi(p) = \pi_1(p) + \pi_2(p) + \ldots + \pi_m(p)$.

(b) There are two periods $t = 1, 2$. In the first period the price vector is p^1 and each firm j chooses some production plan y_j^1. In the second period the price vector is p^2 and each firm j chooses some production plan y_j^2. Assume these choices all satisfy the Weak Axiom of Profit Maximization. Is it always true, possibly true, or never true that the "choices" y^1 and y^2 of the aggregate firm also satisfy WAPM? Explain.

2. George has trouble reading quantity information at the grocery store. For any real-numbered quantity x_1 George only observes the largest integer $I(x_1)$ that is less than or equal to x_1. Similarly, he only observes the largest integer $I(x_2)$ that is less than or equal to x_2. He calculates the utility of a bundle to be $u(x) = \max \{I(x_1), I(x_2)\}$.

(a) Are George's preferences transitive? Continuous? Locally non-satiated? Strongly monotonic? Convex? Briefly explain your answers using a graph.

(b) Consider the optimization problem $\max u(x)$ subject to $p_1x_1 + p_2x_2 \leq m$ and $x \geq 0$. Assume George has no trouble observing prices or income. Does a solution exist? If a solution does exist, is it unique? Can you say whether or not George spends all of his income? Explain.

3. Consider a consumer who only cares about two goods, $x_1 \geq 0$ and $x_2 \geq 0$.

(a) Prove that if the direct utility function is Leontief as a function of the goods, the expenditure function is linear as a function of the prices. Use a graph to explain your reasoning and comment briefly on any important special properties of the Marshallian and Hicksian demands. Note: I am not looking for properties that all demand functions have, just properties resulting from these specific preferences.

(b) Prove that if the direct utility function is linear as a function of the goods, the expenditure function is Leontief as a function of the prices. Use a graph to explain your reasoning and comment briefly on any important special properties of the Marshallian and Hicksian demands. The same note applies as in part (a).

4. There are two ways to produce yogurt (y). Method 1 uses labor (L) and machines (K) according to the production function $y = \min\{aL, bK\}$ where $a > 0$ and $b > 0$. Method 2 uses only labor and has the production function $y = L^{1/2}$. The number of firms in the yogurt industry is fixed at n. The price of yogurt is p, the price of labor is w, and the price of machines is r. Each firm can choose to use either production method, or it can use both methods simultaneously if it wishes.

(a) In the short run each firm has one machine. This is a fixed input but labor can be varied. Solve for the short run supply function $y_i(p)$ of a typical firm i, show this supply function on a graph, and explain verbally how the firm's output changes as the price p rises. Then draw a second graph showing the market supply curve. Be as specific as possible in labeling your graphs.

(b) In the long run again there are n firms (entry is restricted) but machines are now variable (treat them as a continuous input). Draw a graph of the long run market supply curve and justify your answer. What is the maximum possible long run equilibrium price p? Could the equilibrium price be less than this? What would happen if free entry were allowed?

5. There are consumers $i = 1 .. n$ and goods $j = 1 .. k$. Consumer i has the utility function $u_i(x_i) = b_{i1} \ln x_{i1} + b_{i2} \ln x_{i2} + .. + b_{ik} \ln x_{ik}$ where $b_{ij} > 0$ for all j. There is an aggregate endowment vector $w = (w_1 .. w_k) > 0$ for the goods.

(a) A social planner wants to allocate the goods to maximize a function of the form $a_1u_1(x_1) + a_2u_2(x_2) + .. + a_nu_n(x_n)$ where $a_i > 0$ for all i. Solve for the optimal allocation as a function of the parameters a, b, and w.

(b) How could you achieve the same allocation as a Walrasian equilibrium? Say what prices $p_1 .. p_k$ and individual endowments $w_1 .. w_n$ you would use, show that all of the requirements for Walrasian equilibrium are satisfied, and show that the resulting allocation is the one described in part (a).

6. Robinson Crusoe likes leisure (L) and coconuts (C). Crusoe is not endowed with any coconuts but he can obtain them according to the production function $C = H^\alpha$ where H is hours of work and $0 < \alpha < 1$. His utility function is $u(L, C) = L^\beta C^{1-\beta}$ where $0 < \beta < 1$. Crusoe must obey the time constraint $H + L = 1$.

(a) Solve for Crusoe's optimal allocation (L^*, C^*) and show it on a graph.

(b) Suppose there are markets for coconuts and labor, where p is the price of coconuts and w is the wage. Find a price vector (p, w) that supports the allocation in (a) as a Walrasian equilibrium. Use a graph to explain why all of the requirements of WE are satisfied. Does the firm have positive profit in equilibrium? Why or why not?